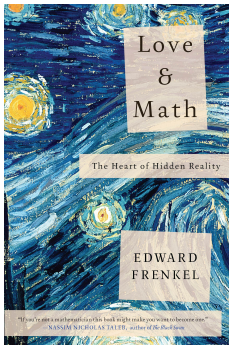
Love & Math, The heart of hidden reality, by *Edward Frenkel*. Basic Books, 2013, ISBN 978-0-465-05074-1 (hbk), 292 pp.

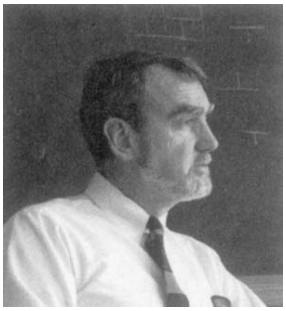


Edward Frenkel

Ever since he grew up as a boy in Kolomna (Russia), Frenkel was fascinated by elementary particles and quantum physics. It was pointed out to him that to understand these, he should start learning mathematics. So he started reading mathematics in his free time. An obvious choice would be to study at the department of Mechanics and Mathematics (*Mekh-Mat*) of the Moscow State University (MGU). However, back in 1984, his father being Jewish, this was made impossible by the ruling anti-semitism. So he registered at the Institute of Oil and Gas (*Kerosinka*), his second choice. However he regularly climbed the fence and sneaked into the MGU to attend some courses and seminars by Gelfand. Besides he worked on a problem of braid groups proposed by D. Fuchs which resulted in his first paper published in *Funct. Anal. Appl.* at the age of 20.

Kerosinka (Gubkin Univ. of Oil & Gas)

André Weil



Robert Langlands

This brought him to study symmetry, (braid) groups, and curves over finite fields. Further work introduced him straight into the *Langlands Program* proposed by *Robert Langlands* in 1967 and more formally in 1970. It is based on an earlier idea of *André Weil* who, while imprisoned in 1940 (having a disagreement with the French authorities), wrote a letter to his sister explaining the idea of a mathematical *Rosetta Stone* which would allow to translate results from three seemingly different fields in mathematics into each other: number theory, curves over finite fields, and Riemann surfaces. Exploring this connection has been shown successful by the proof of *Fermat's Last Theorem*. This connection is the mathematical analog of what the theoretical physicist call the *Grand Unifying Theory* in their study of quantum physics.

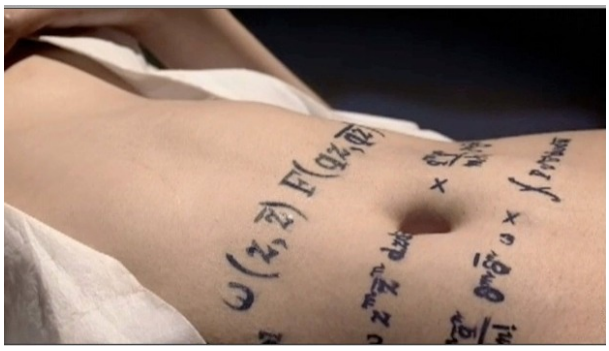
B. Feigin

The mathematical or physical aspects are just two different interpretation of the same theory. So quantum physics is like a fourth column to be added to Weil's *Rosetta Stone*. Frenkel's work with B. Feigin on Kac-Moody algebras came just in time because he got an invitation to spend a semester at Harvard in 1989 at the very time that *perestroika* was emerging. Because of the worsening situation in Russia with an unclear outcome, he decided after his 3 months stay, that it was better not to return to Russia. So he stayed at Harvard where he got his PhD in 1991. Later he became professor of mathematics at UC Berkley. In 2003 he got directly involved in a multi-million DARPA grant to work out more elements of Weil's *Rosetta Stone*. Since then, his mathematical career is largely devoted to building the bits and pieces of this Grand Unifying Theory.

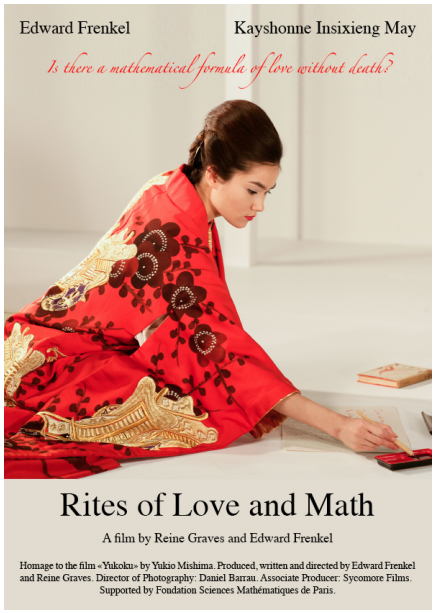
Frenkel makes it crystal clear that he is a passionate lover of mathematics and that his enthusiasm for the *Langlands Program* is immense. This love and passion is what he wants to convey

to the reader. The math that most people learn in school is like learning to paint a fence in an art class, while true painting is about creating master pieces like Da Vinci or Picasso did. Mathematics is also a moral duty. Our world is ruled by mathematics that are hidden to most of us. The financial crisis in 2008 was caused because mathematics was applied by people that were not controlled in a democratic way. Mainly because our society does not care about mathematics and most people tend to stay away from it as far as possible. Mathematics should not be restricted to the "initiated few" but it should be shared by everybody. There is nothing more democratic than mathematics. There are no patents for formulas, it is a universal language, and a correct formula can only represent truth, the universal truth.

истина (*istina*) = truth



The formula of love

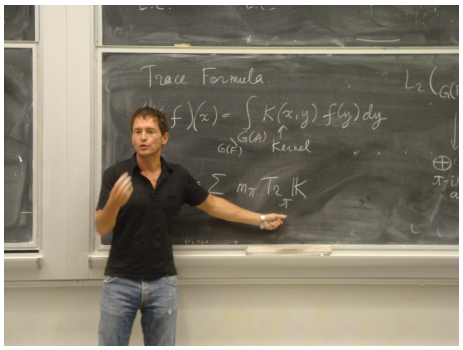


With this conviction, Frenkel wants to transfer not only his love for mathematics but he also wants to show us the beauty of the mathematics that he is devoting most of his life to, and not just the "fence painting" bits. Of course reading this book will not make you a mathematician, but he succeeds to bring his message by describing his life (at least the part related to his mathematical career) and gradually taking the reader along in his conquest of the mathematics he needs. So he explains symmetries, groups, finite fields, SU(3), manifolds, Galois groups, Lie algebras, sheaves, supersymmetry, strings, branes, etc. All things that are far beyond the low-fi kind of math that one usually finds in popular science books. Of course this is not easy, but I can imagine that his charismatic account will make some readers regret that they are not mathematicians, erasing their usual conviction that mathematics is a habitat where only nerds can survive. Many of the more technical details are removed from the main body of the text and deposited as (sometimes quite extensive) notes at the end of the book. For a mathematical reader, they are of course useful, but others may want to skip them and they will still be able to follow the essence of Frenkel's *Conquest of the Holy Grail*.

But Frenkel is not only a mathematician. The last chapter of the book is still about mathematics and love, but now revealing his artistic talents. After a visit to Paris, he got the idea to make a film about math. With his neighbour, the author Thomas Farber, he wrote a screen-play called *The two-body problem* about two men in the South of France, one is a writer, the other a mathematician. They exchange their experiences, their passion for their profession and for women. It was published as a book in 2010. Before starting on the

E. Frenkel and T. Farber

movie project, he wanted to get some cinematographic experience at a smaller scale and decided to produce a short movie. During another visit to France, Reine Graves, a young film director joined in the project. Inspired by a Japanese film of Y. Mishima *Rites of Love and Death* in which a lieutenant commits a ritual suicide together with his wife, Frenkel and Graves imitate the movie more or less. It shows a man (Frenkel) and a women (K.I. May) with in the back a poster with the text *istina* (Russian for truth). The man tattoos a mathematical formula (the formula of love) on the body of the women. The film is called *Rites of Love and Math.* It was well received, and you will find pictures on the Web of Frenkel teaching in Berkeley, but also where he shows up at the Cannes film festival. In fact by different media, Frenkel tries to transmit the same message: a mathematical formula or mathematics in general can be a thrilling thing of beauty, it can give you goose bumps, one may fall in love with it, it represents the ultimate truth, and it is worth committing your life to. The return you get from it is overwhelming.

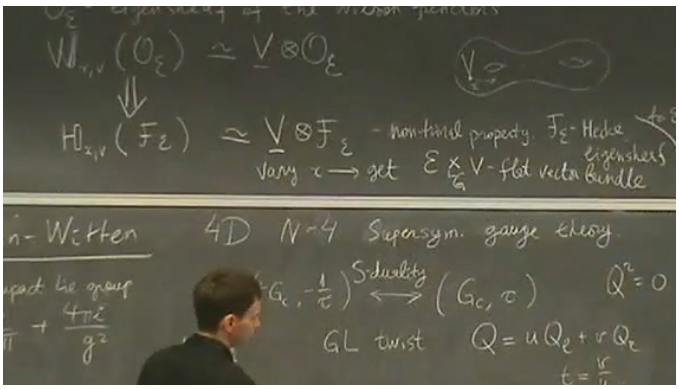


E. Frenkel in Berkley

E. Frenkel in Cannes

Reine Graves

One final remark. It is of course a side remark after Frenkel's plea for beauty, but I do not think that the cover design of the book is a success. It shows text in slightly tilted rectangles on a background image that is a detail of Van Gogh's *Starry Night* painting. The symbolism is obviously well chosen, but it looks terribly chaotic, and I would have preferred a more stylish design representing the mathematical purity and beauty of its contents.



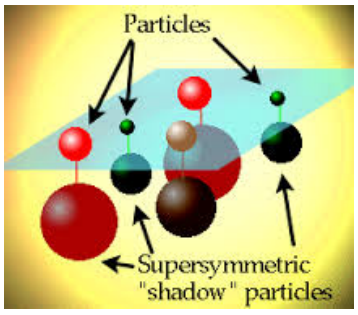
Frenkel lecturing on
Langlands-Type Dualities in Quantum Field Theory

Van Gogh's starry night

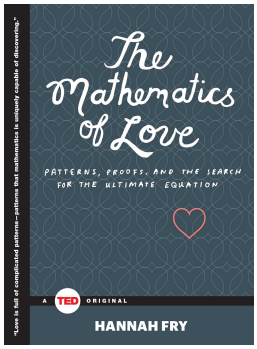
Two of Frenkel's collaborators:

Vladimir Drinfeld (quantum groups)

Edward Witten (quantum field theory supersymmetry)



The Mathematics of Love, by *Hannah Fry*. Simon & Schuster/TED, 2015, ISBN 978-1-476-78488-5 (hbk), 120 pp. TEDx video: `www.youtube.com/watch?v=N37x4GgDVBM`



Hannah Fry

This book is a completely different story, although the titles are similar. TED (Technology, Entertainment, Design) is organizing since 1990 under the slogan *Ideas Worth Spreading* their famous annual conferences in Vancouver, Canada. They are available on their website `www.ted.com`. For example the 2015 lecture was given by *Rajiv Maheswaran* and had the title *The math behind basketball's wildest moves*. But they are not restricted to mathematics. A broad set of cultural, social, and academic subjects are covered in general. The TEDx events are smaller siblings of the general TED talks. They are planned and organized independently by a local community in the usual TED style and using their licence. Hannah Fry's TEDx lecture of March 2014 organized by Binghamton University, NY, was about the *Mathematics of love* and now, one year later, also a related book with the same title is available.

Hannah Fry is a mathematician at the University College London's Centre for Advanced Spatial Analysis and her book follows loosely the talk (which is only about 18 minutes) with some extra chapters added. Since these lectures are for a general public, you cannot expect that deep mathematics are involved. In facts besides showing some spread sheets and a rare graph all she says is that 'some mathematics has been or can be used'.

In the book she goes through the whole process from finding a partner to marriage and living happily ever after. The first chapter is about how many potential candidates you have for courting. By estimating the number of people and taking percentages that satisfy your constraints and preferences, you may end up with some number to choose from. About beauty, she tells not much more than that on facial photographs it is a matter of symmetry, but that in personal contact, slight asymmetries and body language are more important, and that to be chosen it suffices to look only slightly better than somebody standing next to you. Some game theory is involved in selecting a partner when going out with friends and your goal is to maximize the good of the group over the good of the individual. *OKCupid* is a dating site which was started by some mathematicians that use a score of how important you think certain properties are to find a possible match for you. Your popularity on the web depends on how diverse the score is that is attributed to you by others and there are some more ideas on social (and sexual) networks. E.g., an exponential distribution my give an unexpected average. To settle down you have the well known rule that you should reject the 37% first candidates and select the first one after that that is better than all the previous ones. Spreadsheets are advised to select the number of people to invite for the wedding and how you may come up with a distribution of the expected number of people that will actually show up. To arrange people on the tables to avoid rows and optimize the overall happiness (NP hard), you need some software to solve that approximately. Conflicting situations (for example marriage) can be described by a simple dynamic nonlinear system of two coupled difference equations[1]. She gives some very useful common sense advise too, but if you want to learn about the mathematics, you will not be much wiser after reading the book. All you learn is that you can model many things also in a social context, but quotes like 'mathematics learns that...' do not make you per se wiser on a mathematical level.

Adhemar Bultheel

[1]See also *The Mathematics of Marriage* by J.M. Gottman et al, MIT Press, 2005.